CSCI 334: Principles of Programming Languages

Lecture 7: PL Fundamentals III

Instructor: Dan Barowy

Williams

Announcements

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Resubmission procedure

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Wednesday Office Hours now
3pm-5pm
(originally: 10am-noon)

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(If these hours still don't work
for you, make an appointment)

Mental Technique #3

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Confusion is not necessarily a bad thing.

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Sometimes Confusion is a Good Thing
Tania Lombrozo
NPR, December 14, 2015

https://www.npr.org/sections/13.7/2015/12/14/459651340/sometimes-confusion-is-a-good-thing

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Sometimes Confusion is a Good Thing
Tania Lombrozo
NPR, December 14, 2015

"Students who were confused … as reflected in inconsistent responses on subsequent questions … ultimately did better on a final test assessing whether they learned the key points from the lessons."

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Sometimes Confusion is a Good Thing
Tania Lombrozo
NPR, December 14, 2015

"One possibility is that confusion is … a marker that an important cognitive process has taken place: The learner has appreciated some inconsistency or deficit in her prior beliefs. … [A]nother possibility is that confusion is itself a step toward learning — an experience that motivates the learner to reconcile an inconsistency or remedy some deficit. In this view, confusion isn't just a side effect of beneficial cognitive processes, but a beneficial process itself. Supporting this stronger view, there's evidence that experiencing difficulties in learning can sometimes be desirable, leading to deeper processing and better long-term memory."

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Mental Technique #3

The importance of stupidity in scientific research
Martin A. Schwartz
Journal of Cell Science 2008 121: 1771 doi: 10.1242/jcs.033340

"Focusing on important questions puts us in the awkward position of being ignorant. One of the beautiful things about science is that it allows us to bumble along, getting it wrong time after time, and feel perfectly fine as long as we learn something each time. No doubt, this can be difficult for students who are accustomed to getting the answers right."

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It is a signal that you are not confident in your knowledge.

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It is a signal that you are not confident in your knowledge.

Use this signal to guide your study.

Parse Trees

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There are at least two forms of trees
that we might refer to “parse trees”

Derivation Tree

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Describes exactly how input was parsed

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e ::= n | e+e | e-e
n ::= d | nd
d ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
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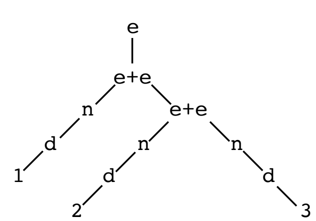

Abstract Syntax Tree

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Abstracts over representation details

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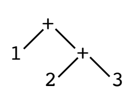

Solution to reduction in reading prompt

```
(λa.λb.(- a b)) 2 1
```

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`(λa.λb.(- a b)) 2 1`

Anyone want to give this a try on the board?

Activity

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`(λf.λx.f(f x))(λz.(+ x z))2`

Activity

Normal order reduction:

`(λf.λx.f(f x))(λz.(+ x z))2`

Activity

Activity

(λf.λx.f(f x))(λz.(+ x z))2

Activity

Applicative order reduction:

(λf.λx.f(f x))(λz.(+ x z))2

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def: a function *f* is **computable** if there
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def: a function *f* is **computable** if there
is a program *P* that computes *f*.

In other words, for **any** (valid) input *x*, the
computation *P(x)* **halts** with output *f(x)*.

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example

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valid inputs are **integers**

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P(x) is:

$f(x) = x + 5$

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computable?
yes, *partially*.

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f(x) = x + 5

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Decide whether program P halts on input x.

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`P(x)` is the output of program `P` run on input `x`.

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How might this work?

Fact: it is provably impossible to write `Halt`

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Finally, derive a fact that contradicts another fact.

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Finally, derive a fact that contradicts another fact.

Therefore, the presupposition must be false.

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Remember: we are looking to produce a contradiction.

The proof is hard to "understand" because the facts it derives don't actually make sense. Don't read too deeply.

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Construct:

DNH(P) = { if Halt(P,P) is true, while(1){}; returns false otherwise }

The Halting Problem: Proof

Suppose:

Halt(P,x) = { returns true if P(x) halts; returns false otherwise } — Halt always halts!

Construct:

DNH does not always halt! — DNH(P) = { if Halt(P,P) is true, while(1){}; returns false otherwise }

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Rewrite:

`DNH(P)` = {
- if `Halt(P,P)` is `true`, `while(1){}`
- returns `false` otherwise

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`DNH(P)` = {
- if `P(P)` halts, run forever
- returns `false` otherwise

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`Foo` is false if `Foo` is true.

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This literally makes no sense.

`Foo` is true if `Foo` is false.
`Foo` is false if `Foo` is true.

Therefore, the `Halt` function cannot exist.

Next class:

How we can use the Halting Problem to show that other problems cannot be solved (in general) by "reduction" to the Halting Problem.

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We cannot tell, in general…

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… if a program eventually produces an error.

… if a program will re-read an item in memory.