CSCI 334:
Principles of Programming Languages

Lecture 17: Polymorphic Type Inference

Instructor: Dan Barowy

Williams

Announcements

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No “assignment”.

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A short reading (Cardelli) for Monday.

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`http://catb.org/jargon/html/F/foo.html`

Project Timeline

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Project checkin: mostly complete by Nov 29

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Project done: complete by Dec 6

Project Timeline

Project checkin: mostly complete by Nov 29

Project done: complete by Dec 6

Project presentation (5-10 minutes): Dec 11

Topics

Topics

Type inference

Topics

Type inference

Project Q & A

Type Inference

type inference

```
let apply f x = f x
```

type inference

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1. convert to λ expression

type inference

```
let apply f x = f x
```

1. convert to λ expression
2. label with type variables

type inference

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let apply f x = f x
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1. convert to λ expression
2. label with type variables
3. generate constraints

type inference

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let apply f x = f x
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1. convert to λ expression
2. label with type variables
3. generate constraints
4. unify

type inference

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let apply f x = f x
```

1. convert to λ expression
2. label with type variables
3. generate constraints
4. unify
5. rename variables

type inference

type inference

Not everybody loves this part of PL.

type inference

Not everybody loves this part of PL.

I hope that you can appreciate the absence of magic.

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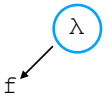

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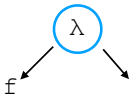

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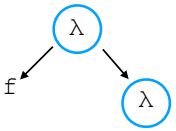

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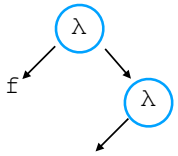

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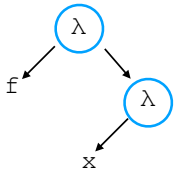

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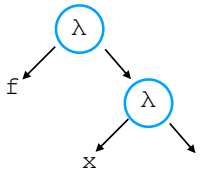

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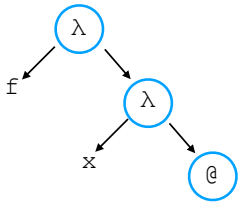

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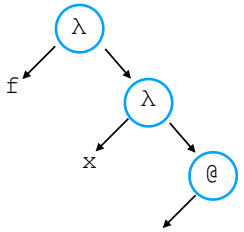

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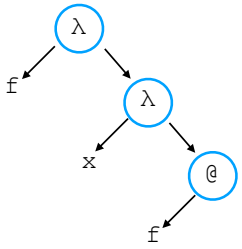

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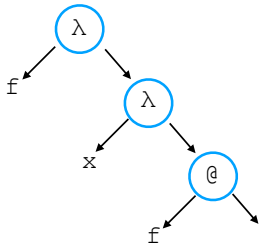

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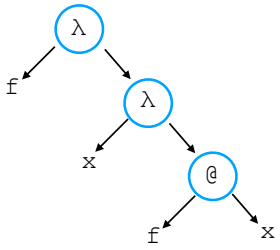

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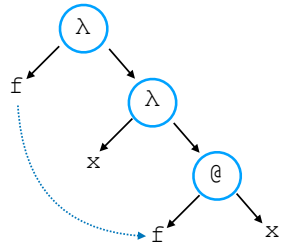

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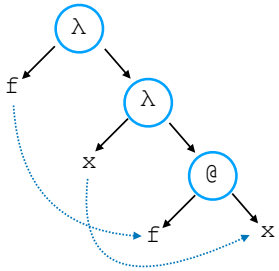

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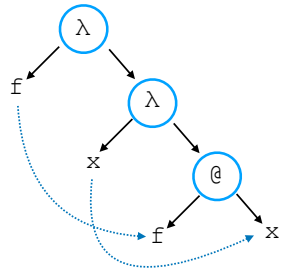

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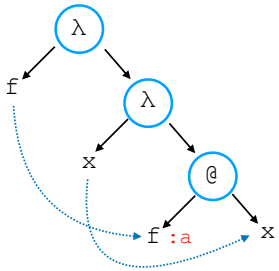

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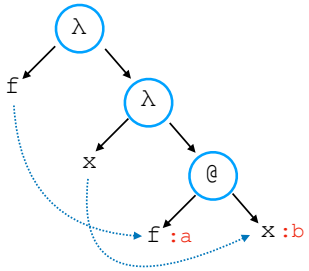

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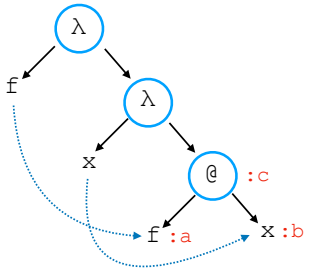

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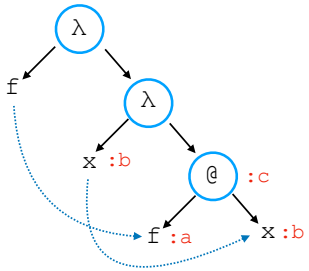

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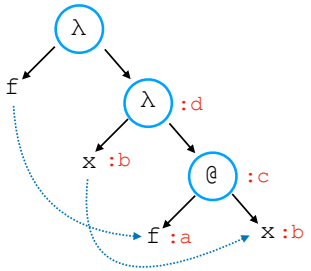

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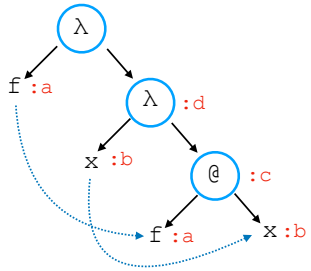

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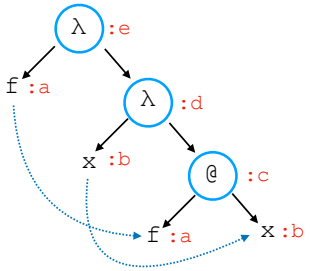

3. generate constraints

`<expr> ::= <var>`	variable
`| λ<var>.<expr>`	abstraction
`| <expr><expr>`	function application

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Three rules, each corresponding to a kind of **λ** expression.

3.1. <var> constraint

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No constraint.

3.2. abstraction constraint

`λ<var>.<expr>`

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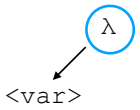

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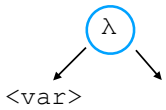

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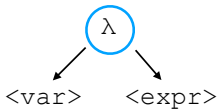

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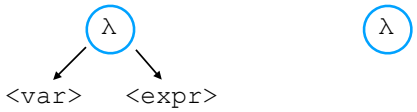

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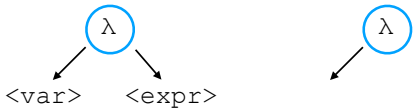

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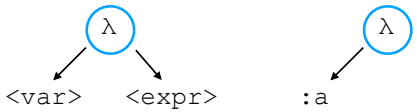

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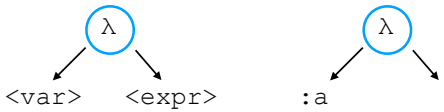

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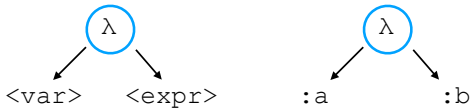

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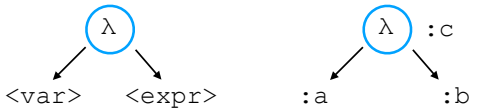

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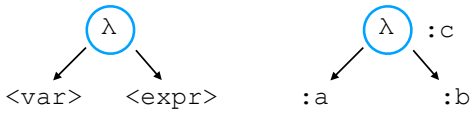

Constraint: If the type of <var> is a and the type of <expr> is b, and the type of λ is c, then the constraint is c = a → b.

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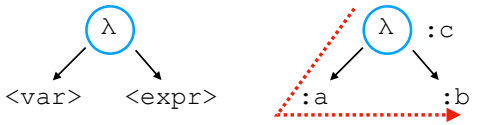

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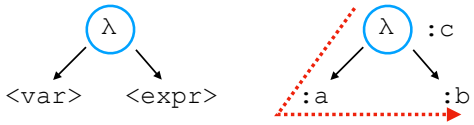
"left triangle rule"



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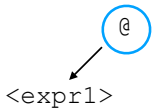

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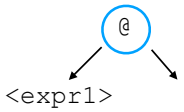

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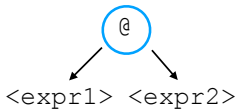

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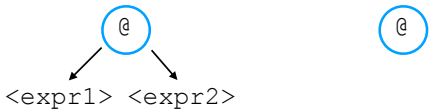

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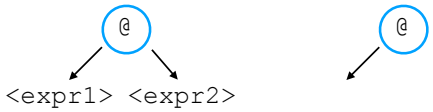

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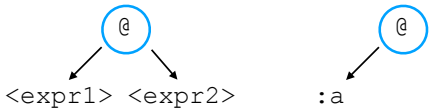

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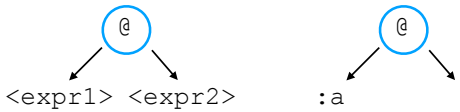

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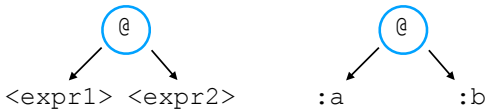

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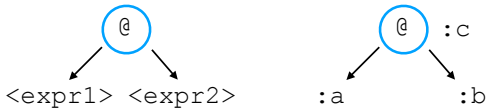

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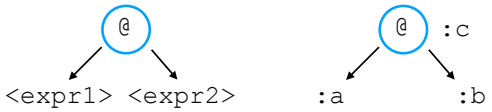

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3. constraints summary

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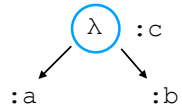
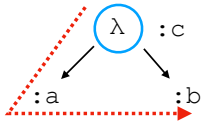


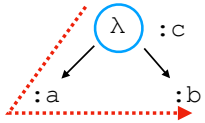
3. constraints summary

Abstraction: If the type of `<var>` is `a` and the type of `<expr>` is `b`, and the type of λ is `c`, then the constraint is `c = a → b`.



3. constraints summary

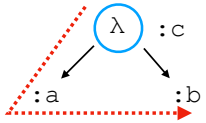
Abstraction: If the type of <var> is a and the type of <expr> is b, and the type of λ is c, then the constraint is c = a → b.



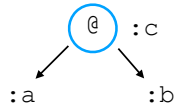
Application: If the type of <expr1> is a and the type of <expr2> is b, and the type of @ is c, then the constraint is a = b → c.

3. constraints summary

Abstraction: If the type of <var> is a and the type of <expr> is b, and the type of λ is c, then the constraint is c = a → b.

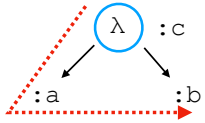


Application: If the type of <expr1> is a and the type of <expr2> is b, and the type of @ is c, then the constraint is a = b → c.

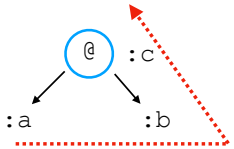


3. constraints summary

Abstraction: If the type of <var> is a and the type of <expr> is b, and the type of λ is c, then the constraint is c = a → b.

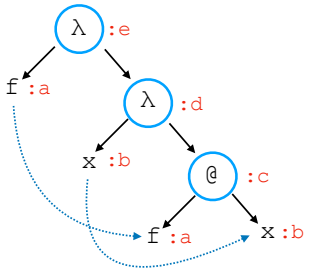


Application: If the type of <expr1> is a and the type of <expr2> is b, and the type of @ is c, then the constraint is a = b → c.

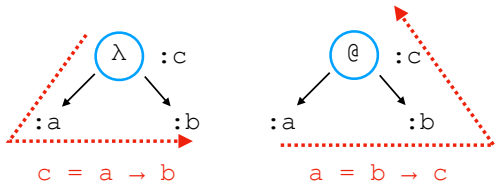


2. label with type variables

```
let apply f x = f x
apply = λf.λx.f x
```

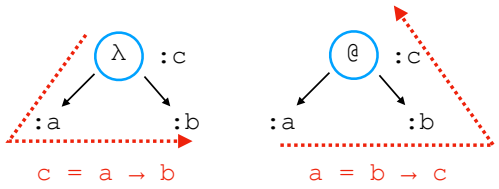


3. generate constraints



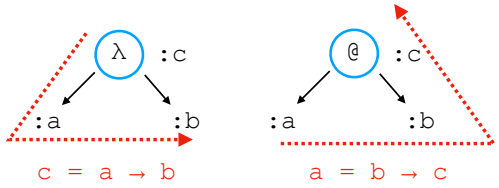
3. generate constraints

subexpression	type	constraint



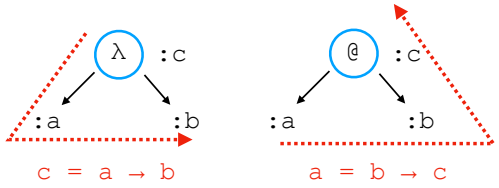
3. generate constraints

subexpression	type	constraint
`f`		



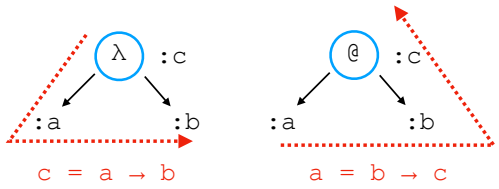
3. generate constraints

subexpression	type	constraint
`f`		
`x`		



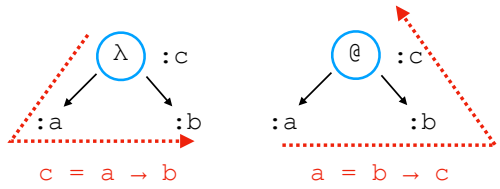
3. generate constraints

subexpression	type	constraint
f		
x		
f x		



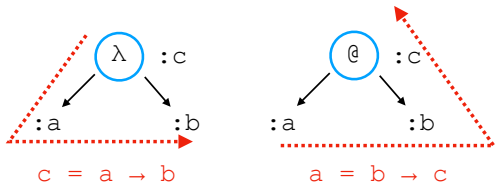
3. generate constraints

subexpression	type	constraint
f		
x		
f x		
λx.f x		



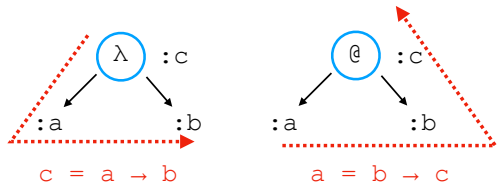
3. generate constraints

subexpression	type	constraint
f		
x		
f x		
λx.f x		
λf.λx.f x		



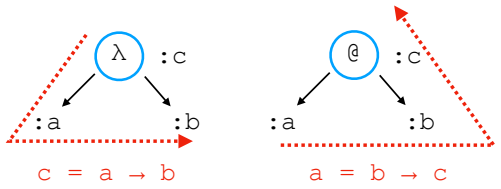
3. generate constraints

subexpression	type	constraint
f	a	
x		
f x		
λx.f x		
λf.λx.f x		



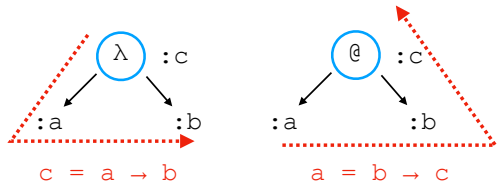
3. generate constraints

subexpression	type	constraint
f	a	
x	b	
f x		
λx.f x		
λf.λx.f x		



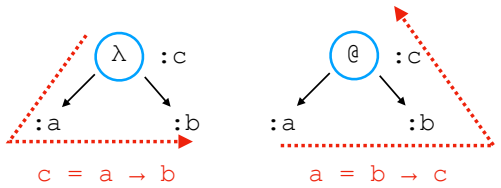
3. generate constraints

subexpression	type	constraint
f	a	
x	b	
f x	c	
λx.f x		
λf.λx.f x		



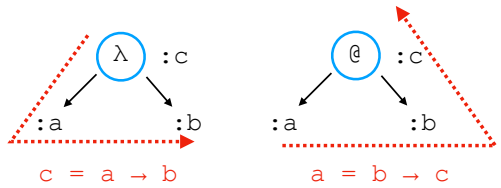
3. generate constraints

subexpression	type	constraint
f	a	
x	b	
f x	c	
λx.f x	d	
λf.λx.f x		



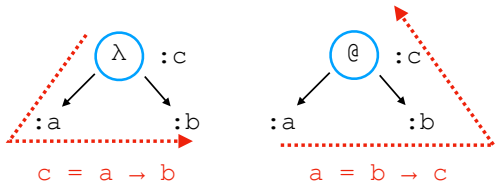
3. generate constraints

subexpression	type	constraint
f	a	
x	b	
f x	c	
λx.f x	d	
λf.λx.f x	e	



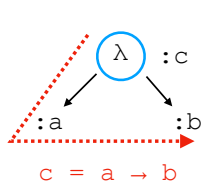
3. generate constraints

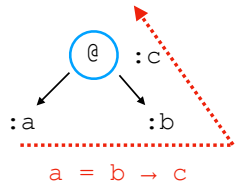
subexpression	type	constraint
f	a	n/a
x	b	
f x	c	
λx.f x	d	
λf.λx.f x	e	



3. generate constraints

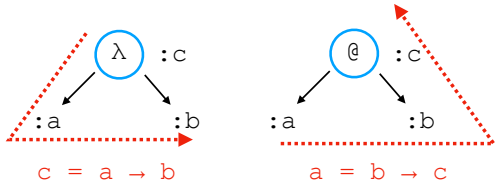
subexpression	type	constraint
f	a	n/a
x	b	n/a
f x	c	
λx.f x	d	
λf.λx.f x	e	





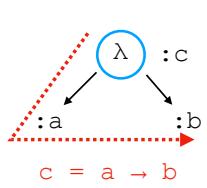
3. generate constraints

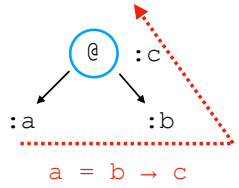
subexpression	type	constraint
f	a	n/a
x	b	n/a
f x	c	a = b → c
λx.f x	d	
λf.λx.f x	e	



3. generate constraints

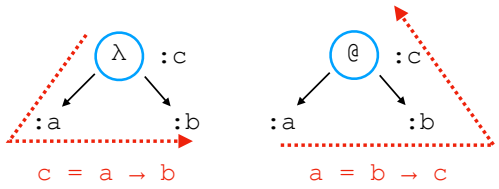
subexpression	type	constraint
f	a	n/a
x	b	n/a
f x	c	a = b → c
λx.f x	d	d = b → c
λf.λx.f x	e	





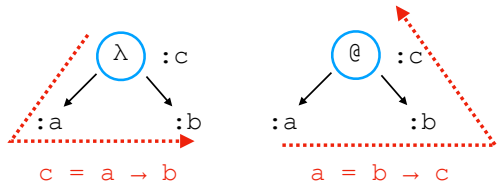
3. generate constraints

subexpression	type	constraint
f	a	n/a
x	b	n/a
f x	c	a = b → c
λx.f x	d	d = b → c
λf.λx.f x	e	e = a → d



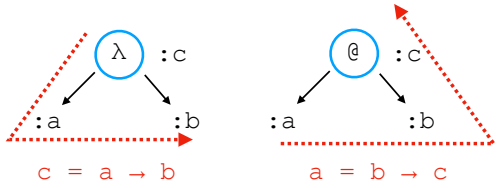
4. unify

subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	d	d = b → c
λf.λx.f x	e	e = b → c → d



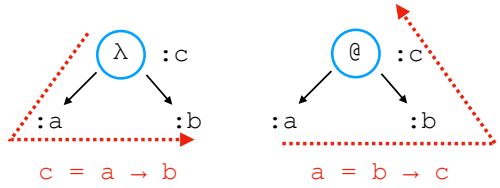
4. unify

subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	b → c	
λf.λx.f x	e	e = b → c → b → c



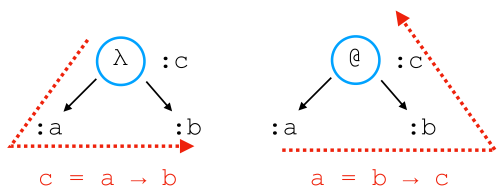
4. unify

subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	b → c	
λf.λx.f x	b → c → b → c	



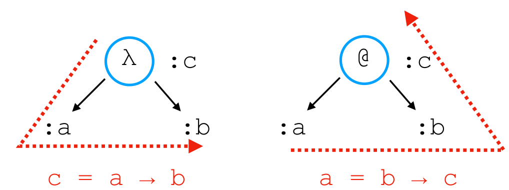
5. rename variables in alpha order

subexpression	type	constraint
f	'a → c	n/a
x	'a	n/a
f x	c	
λx.f x	'a → c	
λf.λx.f x	'a → c → 'a → c	



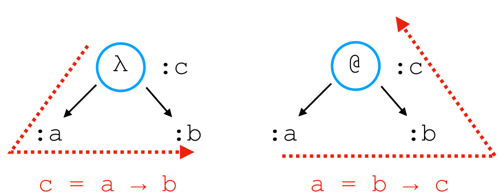
5. rename variables in alpha order

subexpression	type	constraint
f	'a → 'b	n/a
x	'a	n/a
f x	'b	
λx.f x	'a → 'b	
λf.λx.f x	'a → 'b → 'a → 'b	



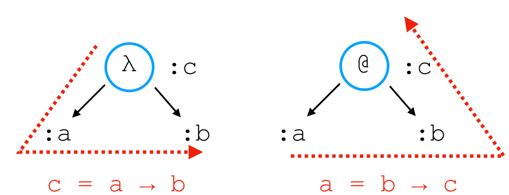
5. rename variables in alpha order

subexpression	type	constraint
f	'a → 'b	n/a
x	'a	n/a
f x	'b	
λx.f x	'a → 'b	
λf.λx.f x	'a → 'b → 'a → 'b	



5. rename variables in alpha order

subexpression	type	constraint
f	'a → 'b	n/a
x	'a	n/a
f x	'b	
λx.f x	'a → 'b	
λf.λx.f x	'a → 'b → 'a → 'b	



Is this the right answer?

`'a → 'b → 'a → 'b`

Is this the right answer?

`'a → 'b → 'a → 'b`

Is this the right answer?

`'a → 'b → 'a → 'b`

```
> let apply f x = f x;;
```

Is this the right answer?

`'a → 'b → 'a → 'b`

```
> let apply f x = f x;;
val apply : f:('a -> 'b) -> x:'a -> 'b
```

Is this the right answer?

`'a → 'b → 'a → 'b`

```
> let apply f x = f x;;
val apply : f:('a -> 'b) -> x:'a -> 'b
```

Lookin' good!

activity

```
let f g x = g (g x)
```

Project Q & A

Next week

Next week

"programming in the large"

Next week

"programming in the large"

object-oriented programming