CSCI 334:
Principles of Programming Languages

Lecture 17: Polymorphic Type Inference

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Williams

Announcements

No “assignment”.

A short reading (Cardelli) for Monday.

Otherwise, work on your project.

Next week: “programming in the large”

`http://catb.org/jargon/html/F/foo.html`

Project Timeline

Project checkin: mostly complete by Nov 29

Project done: complete by Dec 6

Project presentation (5-10 minutes): Dec 11

Topics

Type inference

Project Q & A

Type Inference

type inference

```
let apply f x = f x
```

1. convert to **λ** expression
2. label with type variables
3. generate constraints
4. unify
5. rename variables

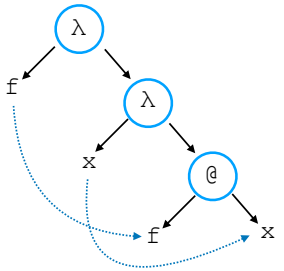
type inference

Not everybody loves this part of PL.

I hope that you can appreciate the absence of magic.

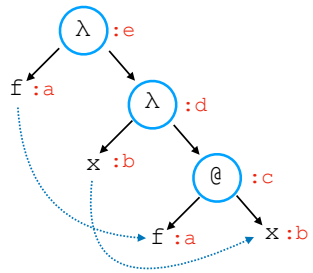
1. convert to **λ** expression

```
let apply f x = f x
apply = λf.λx.f x
```



2. label with type variables

```
let apply f x = f x
apply = λf.λx.f x
```



3. generate constraints

```
<expr> ::= <var>          variable
    |   λ<var>.<expr>     abstraction
    |   <expr><expr>      function application
```

Three rules, each corresponding to a kind of **λ** expression.

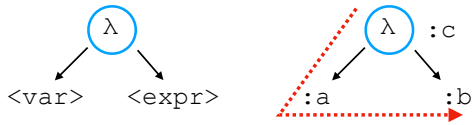
3.1. <var> constraint

No constraint.

3.2. abstraction constraint

`λ<var>.<expr>`

"left triangle rule"

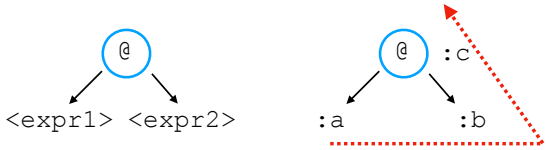


Constraint: If the type of `<var>` is `a` and the type of `<expr>` is `b`, and the type of `λ` is `c`, then the constraint is `c = a → b`.

3.3. application constraint

<expr><expr>

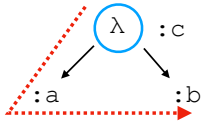
"right triangle rule"



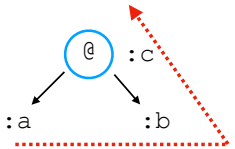
Constraint: If the type of <expr1> is a and the type of <expr2> is b, and the type of @ is c, then the constraint is a = b → c.

3. constraints summary

Abstraction: If the type of <var> is a and the type of <expr> is b, and the type of λ is c, then the constraint is c = a → b.

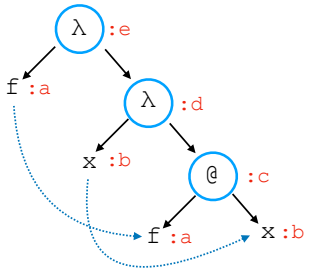


Application: If the type of <expr1> is a and the type of <expr2> is b, and the type of @ is c, then the constraint is a = b → c.



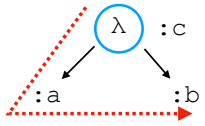
2. label with type variables

```
let apply f x = f x
apply = λf.λx.f x
```

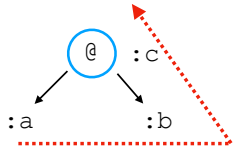


3. generate constraints

subexpression	type	constraint
f	a	n/a
x	b	n/a
f x	c	a = b → c
λx.f x	d	d = b → c
λf.λx.f x	e	e = a → d



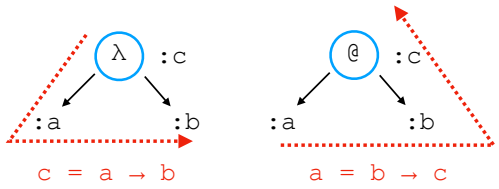
c = a → b



a = b → c

4. unify

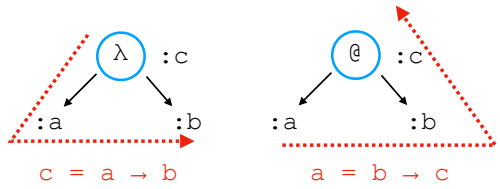
subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	d	d = b → c
λf.λx.f x	e	e = b → c → d



c = a → b a = b → c

4. unify

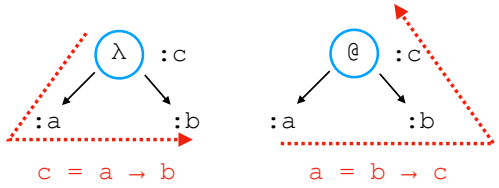
subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	b → c	
λf.λx.f x	e	e = b → c → b → c



c = a → b a = b → c

4. unify

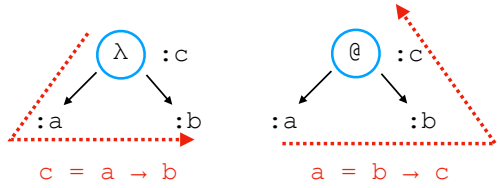
subexpression	type	constraint
f	b → c	n/a
x	b	n/a
f x	c	
λx.f x	b → c	
λf.λx.f x	b → c → b → c	



c = a → b a = b → c

5. rename variables in alpha order

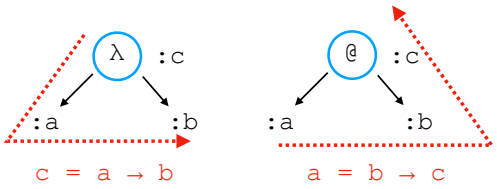
subexpression	type	constraint
f	'a → c	n/a
x	'a	n/a
f x	c	
λx.f x	'a → c	
λf.λx.f x	'a → c → 'a → c	



c = a → b a = b → c

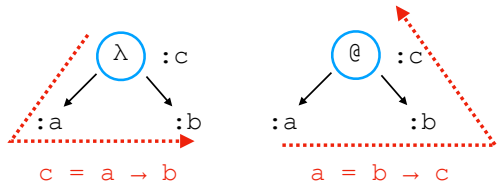
5. rename variables in alpha order

subexpression	type	constraint
f	'a → 'b	n/a
x	'a	n/a
f x	'b	
λx.f x	'a → 'b	
λf.λx.f x	'a → 'b → 'a → 'b	



5. rename variables in alpha order

subexpression	type	constraint
f	'a → 'b	n/a
x	'a	n/a
f x	'b	
λx.f x	'a → 'b	
λf.λx.f x	'a → 'b → 'a → 'b	



Is this the right answer?

'a → 'b → 'a → 'b

```
> let apply f x = f x;;
val apply : f:('a -> 'b) -> x:'a -> 'b
```

Lookin' good!

activity

```
let f g x = g (g x)
```

Project Q & A

Next week

"programming in the large"

object-oriented programming